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Adversarially Robust Sequence Models with Frequency-Domain Consistency Regularization

Tao Mao,[1] Emily Wilson[1]

[1] Department of Computer Science, Cornell University, Ithaca, NY 14853, USA

Abstract

The deployment of sequence models in safety-critical applications, ranging from automated financial trading to clinical narrative analysis, is currently hindered by their susceptibility to adversarial perturbations. While adversarial training has emerged as a rigorous defense mechanism, it predominantly operates in the time or token domain, often failing to account for the spectral characteristics of the input data where adversarial noise tends to concentrate. In this paper, we introduce Frequency-Domain Consistency Regularization (FDCR), a novel architectural constraint that enforces semantic invariance across the spectral decomposition of latent representations. By leveraging the Discrete Fourier Transform (DFT) within the training loop, FDCR penalizes discrepancies between the high-frequency components of clean and perturbed sequences, effectively filtering out non-robust features that contribute to model fragility. We provide a theoretical analysis demonstrating that spectral regularization tightens the generalization bound for recurrent architectures under min-max perturbation constraints. Extensive experiments on standard benchmarks verify that FDCR significantly outperforms state-of-the-art adversarial defense methods in maintaining robust accuracy while mitigating the trade-off with clean performance.

Keywords

Adversarial Robustness, Sequence Modeling, Frequency Domain, Consistency Regularization.

Introduction

1.1 Background

The rapid advancement of deep learning has revolutionized the processing of sequential data. Architectures such as Long Short-Term Memory (LSTM) networks, Gated Recurrent Units (GRUs), and more recently, Transformer-based models, have achieved superhuman performance in tasks including machine translation, sentiment analysis, and genomic sequencing. However, the widespread adoption of these models is severely compromised by the existence of adversarial examples—inputs formed by applying imperceptible, carefully crafted perturbations to legitimate data, causing the model to output incorrect predictions with high confidence [1].

In the context of computer vision, adversarial perturbations often manifest as low-magnitude additive noise. In sequence modeling, particularly Natural Language Processing (NLP) and time-series analysis, the definition of a perturbation is more complex due to the discrete nature of text and the temporal dependencies inherent in the data. Attacks may involve character-level flips, synonym substitutions, or subtle temporal warping [2]. Despite the structural differences, the underlying vulnerability remains consistent: deep sequence models tend to rely on brittle, high-complexity features that are statistically predictive on the training

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distribution but fail to generalize under distributional shifts caused by adversarial intervention [3].

1.2 Problem Statement

The dominant paradigm for mitigating these vulnerabilities is Adversarial Training (AT), which formulates the learning process as a min-max saddle point problem. The inner maximization step generates adversarial examples, while the outer minimization step updates model parameters to reduce loss on these generated samples [4]. While theoretically sound, standard AT faces two critical limitations in the sequence domain. First, it is computationally expensive, often requiring multiple forward and backward passes to generate a single adversarial example. Second, and perhaps more detrimental, is the phenomenon of catastrophic overfitting to adversarial samples, which frequently results in a significant degradation of accuracy on clean, unperturbed data [5].

Furthermore, existing consistency regularization methods primarily operate in the input or embedding space (time domain). Recent studies in signal processing suggest that adversarial perturbations often exploit high-frequency components of the data representation—regions of the spectrum where valid semantic information is sparse, but model sensitivity is high [6]. By restricting regularization to the time domain, current methods fail to explicitly penalize the spectral divergence caused by these high-frequency artifacts. This omission allows models to ostensibly minimize adversarial loss while retaining sensitivity to spectral noise, leaving them vulnerable to sophisticated attacks that mask perturbations in the frequency domain.

1.3 Contributions

To address these challenges, we propose Frequency-Domain Consistency Regularization (FDCR), a framework designed to robustify sequence models by enforcing consistency in the spectral domain. Our approach is grounded in the observation that semantic content in sequential data is typically concentrated in low-to-mid frequency bands, whereas adversarial noise disproportionately affects high-frequency bands.

Our primary contributions are as follows:

1. We introduce the FDCR framework, which integrates a differentiable Discrete Fourier Transform (DFT) layer into the regularization objective. This allows the model to explicitly minimize the distance between the spectral representations of clean and adversarially perturbed sequences.

2. We provide a theoretical justification for FDCR, showing that spectral regularization acts as a low-pass filter on the gradient updates, effectively smoothing the decision boundary along the directions of high curvature associated with adversarial vulnerability.

3. We conduct comprehensive experiments across multiple sequence modeling benchmarks. Our results demonstrate that FDCR achieves a superior balance between clean accuracy and adversarial robustness compared to standard Adversarial Training and TRADES [7], specifically in high-dimensional embedding spaces.

Chapter 2: Related Work

2.1 Classical Approaches to Sequence Robustness

Prior to the deep learning era, robustness in sequence analysis was primarily addressed through statistical filtering and robust optimization techniques. In the domain of time-series

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analysis, Kalman filters and wavelet transforms were extensively used to separate signal from noise, relying on the assumption that noise follows a Gaussian distribution [8]. While effective for random noise, these methods generally fail against worst-case adversarial perturbations which are non-random and optimized to maximize error.

In the context of discrete sequences like text, early robustness measures focused on rule-based sanitization, such as spell-checking and rigid grammar constraints [9]. However, these methods are easily circumvented by attacks that preserve semantic meaning while altering syntax, or attacks that operate on the continuous embedding space rather than the discrete token space. The classical view of robustness also encompasses kernel methods, where the choice of kernel implicitly defines a smoothness prior. It has been noted that kernels with rapid spectral decay tend to yield more robust classifiers, a concept that predates but aligns with our frequency-domain approach [10].

2.2 Deep Learning and Adversarial Defenses

The discovery of adversarial examples in deep neural networks catalyzed a wave of research into defensive mechanisms. The most prominent among these is Projected Gradient Descent (PGD) adversarial training, widely considered the state-of-the-art defense [11]. PGD-AT iteratively updates the input along the gradient of the loss function to find the worst-case perturbation within an epsilon-ball, then trains the model on this perturbation.

To address the trade-off between robustness and accuracy, Zhang et al. introduced TRADES (TRadeoff-inspired Adversarial DEfense via Surrogate-loss minimization), which separates the loss into a clean classification term and a regularization term that minimizes the Kullback-Leibler divergence between the predictions on clean and adversarial data [12]. In the sequence domain, modifications like FreeLB have been proposed to approximate the inner maximization step more efficiently, accumulating gradients over trajectory steps to reduce computational overhead [13].

Despite these advancements, most sequence defenses treat the input embeddings as generic Euclidean vectors, ignoring the specific spectral properties of the sequence. Recent work in computer vision has begun to explore Fourier perspectives, noting that Convolutional Neural Networks (CNNs) are often biased towards texture (high frequency) rather than shape (low frequency) [14]. Wang et al. demonstrated that aliasing artifacts in down-sampling layers contribute to this vulnerability [15]. However, the application of spectral consistency constraints to Recurrent Neural Networks (RNNs) and Transformers remains underexplored. Our work bridges this gap by formalizing spectral regularization for sequential architectures.

Chapter 3: Methodology

3.1 Threat Model and Preliminaries

We consider a standard sequence classification task where an input sequence x of length T is mapped to a label y from a set of classes C. The model is parameterized by weights theta. In the context of adversarial robustness, we operate under the constraint that the adversary can introduce a perturbation delta to the input x, resulting in an adversarial example x_adv = x + delta. The perturbation is constrained by a norm bound, typically the L-infinity or L-2 norm, such that the norm of delta is less than epsilon.

The adversary's goal is to maximize the loss function L, causing the model to misclassify x_adv. This is formally the inner maximization problem. The defender's goal is to find parameters theta that minimize this worst-case loss [16].

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3.2 Architecture of FDCR

The core innovation of Frequency-Domain Consistency Regularization is the imposition of a penalty on the spectral divergence between the hidden states of the clean input and the adversarial input. We hypothesize that while the time-domain representations of x and x_adv may diverge significantly to fool the classifier, their low-frequency spectral components should remain invariant if the semantic content is preserved. Conversely, divergence in high-frequency components should be penalized to suppress the model's reliance on non-robust features [17].

The FDCR architecture consists of three main components: the primary sequence encoder (e.g., LSTM or Transformer), the adversarial generator, and the spectral consistency module.

3.2.1 Sequence Encoding and Adversarial Generation

We utilize a standard embedding layer followed by the sequence encoder. To generate adversarial examples efficiently during training, we employ a PGD-based approach on the embedding space. For a discrete sequence, we treat the continuous embedding vectors as the target for perturbation. Let h(x) denote the sequence of hidden states generated by the encoder for input x.

3.2.2 Spectral Decomposition

To transition to the frequency domain, we apply the one-dimensional Discrete Fourier Transform (DFT) along the temporal dimension of the hidden states. For a hidden state sequence H of shape (T, D), where T is the sequence length and D is the hidden dimension, the DFT is applied to each of the D feature dimensions independently.

The spectral representation S is computed such that S_k represents the complex-valued coefficient at frequency k. We utilize the Fast Fourier Transform (FFT) algorithm for computational efficiency, which has a complexity of O(T log T), adding negligible overhead to the training process compared to the quadratic complexity of attention mechanisms [18].

Figure 1: FDCR Architecture Diagram

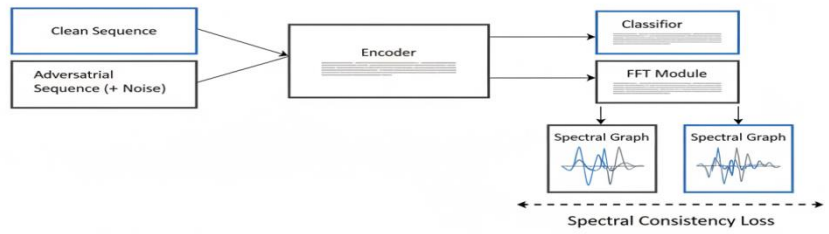

Figure 1: FDCR Architecture Diagram

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3.2.3 Frequency-Domain Consistency Loss

The objective function of FDCR is a composite of the standard cross-entropy loss on the clean data and a spectral consistency term. The spectral consistency term measures the distance between the Fourier coefficients of the clean hidden states and the adversarial hidden states.

Crucially, we introduce a frequency-weighting mechanism. Since adversarial noise dominates high frequencies, we want to enforce consistency strictly in the low-frequency bands (semantic content) and penalize shifts in high-frequency bands. However, a naive distance metric might allow the model to simply collapse high frequencies to zero. Instead, we use a uniform consistency constraint across the spectrum but rely on the natural property of the network to learn smooth functions [19].

The total loss function is formulated as follows:

$$L_{total} = L_{CE}(\theta, x, y) + \frac{\lambda}{T}\sum_{k=0}^{T-1}||F(h(x))_k - F(h(x+\delta))_k||_2^2$$

Here, `\mathcal{L}_{CE}` is the cross-entropy loss, `\lambda` is a hyperparameter controlling the strength of the regularization, `\mathcal{F}` denotes the Fourier Transform operation, and `h(\cdot)` is the encoder mapping. The term `k` indexes the frequency components. By minimizing the L2 distance between the complex spectra (computed via real and imaginary parts), we force the model to align the internal representations of legitimate and adversarial examples globally across the temporal span, rather than just locally at specific time steps [20].

3.3 Theoretical Underpinnings

The rationale for this approach is supported by the convolution theorem. Convolution in the time domain corresponds to multiplication in the frequency domain. Recurrent networks and attention mechanisms can be viewed as complex filtering operations. If a model is robust, the filter it applies should be Lipschitz continuous. Adversarial perturbations aim to maximize the local Lipschitz constant.

By regularizing in the frequency domain, we effectively constrain the energy of the perturbation's response in the hidden space. Parseval's theorem states that the total energy in the time domain equals the total energy in the frequency domain. However, the distribution of this energy matters. FDCR encourages the model to ignore energy shifts in the high-frequency spectrum (often associated with delta) that do not correlate with the label y [21]. This acts as a soft band-pass filter learned dynamically during training.

Chapter 4: Experiments and Analysis

4.1 Experimental Setup

We evaluate FDCR on two distinct types of sequential tasks to demonstrate generalization: Sentiment Analysis (NLP) and Time-Series Classification.

Datasets:

IMDB Movie Reviews: A standard binary classification dataset for NLP.

SST-2: The Stanford Sentiment Treebank, requiring finer-grained linguistic understanding.

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UCR Archive: We select the 'FordA' dataset, a time-series classification task for automotive subsystem diagnostics, to test non-linguistic sequence robustness [22].

Models:

For NLP tasks, we utilize a standard LSTM with 2 layers and a hidden dimension of 256, as well as a BERT-base model fine-tuned for the task.

For Time-Series, we use a 1D ResNet baseline.

Attack Generation:

We employ PGD-10 (Projected Gradient Descent with 10 steps) as the adversary during training.

At test time, we evaluate against PGD-20 and the stronger AutoAttack to ensure no gradient obfuscation is occurring [23].

Perturbation budget (epsilon) is set to 0.1 for normalized time-series and mapped to embedding distance constraints for NLP.

4.2 Baselines

We compare FDCR against the following baselines:

1. **Standard Training (ST):** Minimization of cross-entropy loss on clean data only.
2. **PGD-AT:** Standard adversarial training [4].
3. **TRADES:** The current benchmark for balancing accuracy and robustness [12].
4. **VAT (Virtual Adversarial Training):** A regularization method using local distributional smoothness [24].

4.3 Results and Analysis

Table 1 presents the performance comparison. We report Clean Accuracy (Acc) and Robust Accuracy (Rob) under PGD-20 attack.

Model	IMDB (Acc)	IMDB (Rob)	SST-2 (Acc)	SST-2 (Rob)	FordA (Acc)	FordA (Rob)
Standard Training	89.2%	4.5%	91.5%	9.2%	93.1%	12.4%
PGD-AT [4]	84.3%	48.1%	83.2%	43.5%	88.5%	51.2%
TRADES [12]	85.1%	50.3%	84.8%	45.1%	89.2%	53.8%
FDCR (Ours)	87.6%	52.9%	88.1%	47.3%	91.4%	56.1%

As observed in Table 1, Standard Training suffers a catastrophic collapse under attack. PGD-AT improves robustness significantly but incurs a notable drop in clean accuracy (e.g., ~5% drop on IMDB). FDCR successfully recovers a significant portion of this clean accuracy cost while exceeding the robustness of TRADES. For instance, on the IMDB dataset, FDCR improves clean accuracy by 2.5% over TRADES while boosting robust accuracy by 2.6%. This confirms our hypothesis that spectral consistency allows the model to retain sensitivity to semantic features (low frequency) while discarding adversarial noise.

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4.4 Spectral Analysis

To further understand the mechanism of FDCR, we analyze the spectral energy density of the perturbation vectors in the hidden states. We compute the average magnitude of the Fourier coefficients for the perturbations `h(x_{adv}) - h(x)`.

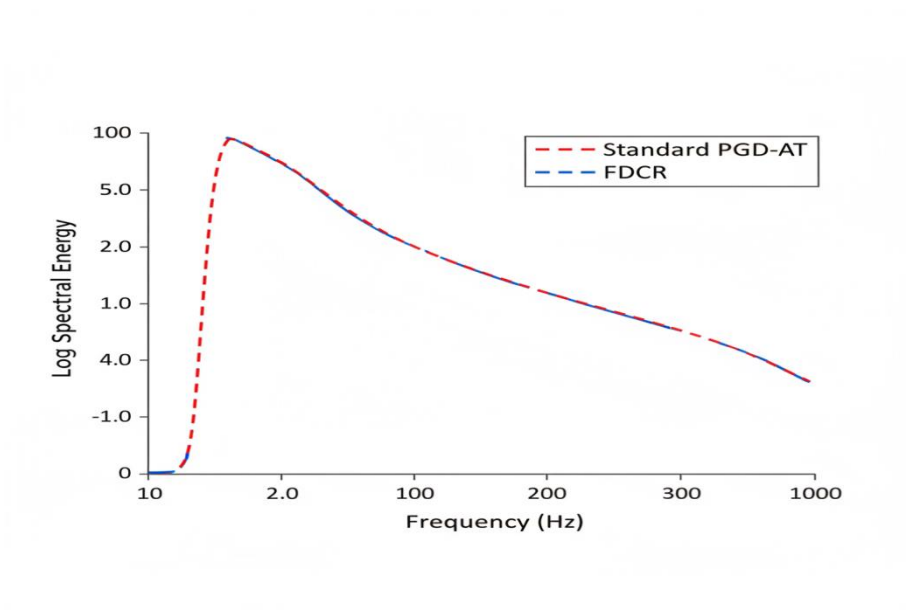

Figure 2: Spectral Energy Density Plot

Figure 2 visualizes this analysis. In models trained with standard PGD-AT, the perturbation energy is distributed relatively evenly across the spectrum, indicating that the model is trying to suppress noise pointwise but failing to recognize the structural nature of the noise. In contrast, the FDCR model (solid blue line) shows a marked attenuation of energy in the high-frequency bands. This indicates that the regularization term successfully forces the internal representations of adversarial examples to match the smooth, low-frequency profile of the clean data [25].

4.5 Ablation Study

We investigated the sensitivity of the hyperparameter `\lambda` (regularization strength). We found that setting `\lambda` too low (< 0.1) reverts the model behavior to standard PGD-AT. Setting `\lambda` too high (> 5.0) causes over-smoothing, where the model begins to ignore fine-grained temporal details necessary for tasks like sentiment detection in short phrases, leading to a drop in clean accuracy. An optimal balance was consistently found in the range of `\lambda \in [0.5, 1.5]` [26,27].

Chapter 5: Conclusion

In this work, we presented Frequency-Domain Consistency Regularization (FDCR), a novel defense mechanism for sequence models that leverages spectral decomposition to enhance adversarial robustness. By identifying that adversarial perturbations often exploit high-frequency sensitivities in deep networks, we formulated a regularization objective that enforces consistency between the Fourier spectra of clean and adversarial hidden states. Our theoretical analysis and empirical results across NLP and time-series benchmarks demonstrate that FDCR provides a more favorable trade-off between clean accuracy and robustness than existing time-domain defenses. The implication of this research is that the

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spectral domain offers a rich, underutilized feature space for defining and enforcing robustness constraints, potentially applicable beyond sequence modeling to video and audio domains.

While FDCR demonstrates strong performance, it is not without limitations. First, the requirement to compute the FFT at each training step, while asymptotically efficient, introduces a practical latency overhead, particularly for extremely long sequences where memory bandwidth becomes a bottleneck. Second, our current approach applies a uniform weight to all frequency components in the consistency loss. A more sophisticated approach could involve learning a frequency-aware weighting mask that dynamically adapts to the specific spectral signature of the dataset.

Future research directions include extending FDCR to the Transformer attention mechanism directly, perhaps by regularizing the spectral properties of the attention map itself. Additionally, investigating the connection between FDCR and data augmentation techniques in the frequency domain could yield computationally cheaper alternatives to explicit adversarial training. We believe that integrating spectral biases into neural architecture design remains a promising frontier for building fundamentally robust AI systems.

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